Dynamic Term-Modal Logic

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ABSTRACT. A first-order dynamic epistemic logic is developed where the names of the agents are also terms in the sense of first-order logic. Consequently one can quantify over epistemic modalities. Using constructs from dynamic logic one can express many interesting concepts. First-order update models are developed and added to the language as modalities.

1 Introduction

The aim of this paper is take a first step in the development of a dynamic epistemic predicate logic. Although there is an extensive literature on dynamic epistemic logic, there are no systems that also involves predicates and quantification. In order to achive this I bring together ideas from term-modal logic, first-order dynamic logic, and dynamic epistemic logic.

Term-modal logic is the title of a paper by Fitting, Thalmann and Voronkov [FTV01], which is closely related to work of Grove and Halpen [GH93, Gro95]. In propositional epistemic logic agents and the names of agents are not distinct (cf. [FHMV95, MvdH95]). The idea of term-modal logic is to treat the agents' names as terms from first-order logic. This is quite natural in a first-order setting, and one can subsequently also quantify over the epistemic modalities associated with the agents, which gives the logic a second-order flavor. This idea was already present right at the start of modern epistemic logic.

> We could develop an alternative system in which the epistemic modalities are treated as "relative" to persons. In this system we should have to deal with expressions like "known to somebody", "unknown to everybody", *etc.* [vW51, p. 35]

This idea is also worked out by Hintikka [Hin62], but not in as much detail as in the work of Fitting et al., Grove, and Halpern. The notation used by

Fitting et al. fits in nicely with the idea that an epistemic interpretation of propositional dynamic logic (PDL) is well-suited for reasoning about information change in multi-agent settings, i.e. to regard PDL as a dynamic epistemic logic, as it is promoted by van Benthem et al. [vBvEK06]. In this paper we extend this idea to first-order dynamic logic.

In this paper these ideas are brought together in the form of a dynamic term-modal logic, which is interpreted epistemically. In Section 2 the language and semantics of this logic is given. In Section 3 we illustrate the expressivity of the language with a number of examples. In Section 4 we define first-order update models. In Section 5 conclusions are drawn and directions for future research are indicated.

2   Language and semantics

First-order dynamic logic was initially developed by Pratt [Pra76] as a modal logic for reasoning about computer processes. The language of dynamic term-modal logic is just the language of first-order dynamic logic (with wildcard assignment), but in this case the set of first-order terms is the same as the set of atomic programs. The terms refer to agents, and the modalities involving these are interpreted epistemically. The language also contains the 'random' or 'wildcard' assignment such that we do not need separate quantifiers and we can easily express common knowledge (see Section 3). For a textbook introduction to first-order dynamic logic including wildcard assignment, see [HKT00, chapter 11]. In the language of dynamic term-modal logic all logical operators except negation, conjunction and identity are dynamic operators.

DEFINITION 1 (language). Let a set of n-ary predicate letters $\mathbf{P}^n$ for each $n \in \mathbb{N}$, a set of n-ary function letters $\mathbf{F}^n$ for each $n \in \mathbb{N}$ and a set of variables $\mathbf{V}$ be given. The language $\mathscr{L}$ is given by the following Backus-Naur Form (where φ are formulas, τ are terms, and α are modalities):

$$
\begin{array}{rcl}
\varphi & ::= & P^n \tau_1 \dots \tau_n \mid \neg\varphi \mid \varphi \wedge \varphi \mid \tau = \tau \mid [\alpha]\varphi \\
\tau & ::= & x \mid f^n(\tau_1, \dots, \tau_n) \\
\alpha & ::= & \tau \mid ?\varphi \mid x := \tau \mid x :=? \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha^*
\end{array}
$$

where $P^n \in \mathbf{P}^n$, $f^n \in \mathbf{F}^n$, and $x \in \mathbf{V}$. We will use the usual abbreviations.

One might think, from the definition above, that the language does not contain constant symbols. We, however, view them as nullary functions, just as propositional atoms are viewed as nullary predicates. In the semantics we will allow for nullary function symbols to be non-rigid, i.e. to refer to different agents in different worlds just as it is argued for by Fitting and Mendelsohn [FM98, chapter 9] in the context of first-order modal logic.

In epistemic contexts it is not unusual that one does not know someone's name. Teachers have to learn the names of their students although they have a complete list of names. They may even know the list by heart, but not which name belongs to which student. This can be modeled by letting constants be non-rigid. Given the epistemic interpretation we give to the language a formula of the form $[\tau]\varphi$ is to be read as 'τ knows that φ'.

In the models we use to interpret this language, the set of agents serves both as the domain of discourse as well as labels for the accessibility relations.

DEFINITION 2 (Epistemic models). An epistemic model M is a tuple (W, A, R, I)

- $W \neq \emptyset$ a non-empty set of possible worlds.

- $A \neq \emptyset$ a non-empty set of agents.

- $R : A \to \wp(W \times W)$ assigns an accessibility relation to each agent.

- I is a function from W to n-ary predicate letters to A^n, and from W to n-ary function symbols to n-ary functions on A.

Note that we take the domain of discourse (the set of agents) to be constant and the interpretation function is dependent upon the world. This is contrary to the models given by Fitting et al. [FTV01]. Note also that in these models there are only agents. In richer models one might have a two-sorted domain with agents and other objects. For the sake of simplicity we have not done so here. Given the epistemic interpretation we give to the logic, we assume that the accessibility relations are equivalence relations.

In order to interpret the three sorted language on these models, we take a world $w \in W$ and – just as in first-order logic – a variable assignment[1] g that assigns an element of A to each variable x. The variable assignment will play a special role in the semantics. The variable assignment does not depend on the world, and hence free variables can be seen as rigid designators. The relations associated with the modalities will be relations on pairs consisting of a world and a variable assignment. We will write $g[x/a]$ for the variable assignment that differs from g at most in what it assigns to x, namely a.

$$g[x/a](y) = \begin{cases} g(y) & \text{if } x \neq y \\ a & \text{otherwise} \end{cases}$$

The idea of viewing assignment change as a modal operator ties in with the idea of viewing first-order logic as a modal logic on assignments as it was

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[1] We will refer to both variable assignment and assignment modalities ($x := \tau$) as assignments.

developed by van Benthem [vB96, chapter 9]. In order to clearly distinguish when a term τ is interpreted as referring to an agent or when it is interpreted as the modality associated with an agent, we use $[\![\tau]\!]_{M,w,g}$ for the former and $[\![\tau]\!]_M$ for the latter.

DEFINITION 3 (semantics). Let an epistemic model $M = (W, A, R, I)$, a world $w \in W$, and an assignment $g : \mathbf{V} \to A$ be given.

$$
\begin{aligned}
M, w, g &\models P\tau_1 \ldots \tau_n & \text{iff} \quad & ([\![\tau_1]\!]_{M,w,g}, \ldots, [\![\tau_n]\!]_{M,w,g}) \in I_w(P^n) \\
M, w, g &\models \neg\varphi & \text{iff} \quad & M, w, g \not\models \varphi \\
M, w, g &\models \varphi \wedge \psi & \text{iff} \quad & M, w, g \models \varphi \text{ and } M, w, g \models \psi \\
M, w, g &\models \tau_1 = \tau_2 & \text{iff} \quad & [\![\tau_1]\!]_{M,w,g} = [\![\tau_2]\!]_{M,w,g} \\
M, w, g &\models [\alpha]\varphi & \text{iff} \quad & M, v, h \models \varphi \text{ for all } v \text{ and } h \\
& & & \text{such that } (w, g)[\![\alpha]\!]_M(v, h)
\end{aligned}
$$

$$
\begin{aligned}
[\![x]\!]_{M,w,g} &= g(x) \\
[\![f^n(\tau_1, \ldots, \tau_n)]\!]_{M,w,g} &= I_w(f^n)([\![\tau_1]\!]_{M,w,g}, \ldots, [\![\tau_n]\!]_{M,w,g})
\end{aligned}
$$

$$
\begin{aligned}
[\![\tau]\!]_M &= \{((v, h), (u, h)) \mid (v, u) \in R([\![\tau]\!]_{M,v,h})\} \\
[\![?\varphi]\!]_M &= \{((v, h), (v, h)) \mid (M, v, h) \models \varphi\} \\
[\![x := \tau]\!]_M &= \{((v, h), (v, k)) \mid k = h[x/[\![\tau]\!]_{M,v,h}]\} \\
[\![x :=?]\!]_M &= \{((v, h), (v, k)) \mid k = h[x/a] \text{ and } a \in A\} \\
[\![\alpha; \alpha']\!]_M &= [\![\alpha]\!]_M \circ [\![\alpha']\!]_M \\
[\![\alpha \cup \alpha']\!]_M &= [\![\alpha]\!]_M \cup [\![\alpha']\!]_M \\
[\![\alpha^*]\!]_M &= [\![\alpha]\!]_M^*
\end{aligned}
$$

Note the interpretation of the modalities does not depend on the actual world, or the assignment, only on the model.

As is also noted by Pratt [Pra76] wildcard assignments can be used as quantifiers. The following formulas $[x :=?]\varphi$ and $\langle x :=?\rangle\varphi$ are clearly equivalent to $\forall x \varphi$ and $\exists x \varphi$ respectively using classical semantics.

Since the domain of discourse (the set of agents) does not vary from world to world the analogue of the Barcan formula holds.

$$[x :=?][\alpha]\varphi \to [\alpha][x :=?]\varphi \tag{1}$$

where x does not occur in α. If $\alpha = \tau$ this formula states that if for all x agent τ knows that φ, then agent τ knows that for all x it holds that φ.

In epistemic logic the accessibility relations are usually taken to be equivalence relations (relations that are reflexive, transitive and symmetric). In propositional modal logic such frame restrictions are captured by axioms. The axiom for transitivity, which is called positive introspection in epistemic logic, is indeed valid for all agents.

$$[x :=?]([x]\varphi \to [x][x]\varphi) \tag{2}$$

However, the rule of universal instantiation is not valid here. So from (2) we cannot deduce that for every term τ that the following formula is valid.

$$[\tau]\varphi \rightarrow [\tau][\tau]\varphi \tag{3}$$

This formula is not valid in the logic proposed above in transitive frames. Take a model M where $W = \{w, v, u\}$, $A = \{a, b\}$, $R(a) = \{w, v\}^2 \cup \{u\}^2$, $R(b) = W^2$. Let $c \in \mathbf{F}^0$. Take $I_w(c) = a$, $I_v(c) = b$. Let $p \in \mathbf{P}^0$. Take $I_w(p) = I_v(p) = 1$ and $I_u(p) = 0$. Let g be an arbitrary assignment. Then $M, w, g \models [c]p$, but $M, w, g \not\models [c][c]p$. This is due to the fact that c is not a rigid designator. A real life example of this is when the worst student knows that Scott is the author of Waverley, but it is not the case that the worst student knows that the worst student knows that Scott is the author of Waverley, assuming the worst student does not know that he is the worst student. (Hintikka gives a similar example about the "greatest fool in Christendom" [Hin62, p. 22].)

The example above shows that universal instantiation is not valid. However the following principle is valid.

$$[x :=?]\varphi \rightarrow [x := \tau]\varphi \tag{4}$$

This principle could be seen as a weaker version of universal instantiation. Applying it to the formula above yields $[x := \tau]([x]\varphi \rightarrow [x][x]\varphi)$, which is indeed also valid.

The complexity of the validity problem of dynamic term-modal logic is Π_1^1 hard, because the validity problem for first-order dynamic logic is Π_1^1 complete (see [HKT00, chapter 13]), and we can translate any validity problem of first-order dynamic logic to a validity problem of dynamic term-model logic. So we cannot hope for a complete finitary proof system. Even if we only take the monadic fragment the logic is still undecidable, given the results of Kripke [Kri62].

3 Expressivity

In this section I will show how several important concepts in first-order modal logic and epistemic logic can be expressed using the logic introduced in the previous section.

In first-order modal logic the distinction between *de dicto* and *de re* is usually illustrated using sentences that involve a modality and a quantifier. Quine [Qui56] considers the sentence 'Ralph believes that someone is a spy', which can be read as 'Ralph believes that there are spies' and 'There is someone whom Ralph believes to be a spy'. These are the *de dicto* and the *de re* reading respectively. The Latin terms were introduced in medieval

logic where modalities were either seen to apply to propositions or modalities were part of predicates applied to subjects [KK84, p. 236]. It is important to clearly distinguish these in any first-order modal logic.

A modern treatment of this distinction can be found in the work of Fitting and Mendelsohn [FM98], which is based on the work of Stalnaker and Thomason [ST68, TS68]. Here, a technique called predicate abstraction is used to make new predicates that may involve modalities. This technique stems from lambda calculus. If $\varphi(x)$ is a formula with a free variable x, then $\langle \lambda x.\varphi(x) \rangle$ is a unary predicate. In this way a distinction can be made between *de dicto* and *de re* predication.

Two readings of 'The number of planets is necessarily odd' can be distinguished. The *de dicto* reading can be paraphrased as 'It is necessary that the number of planets is odd', and the *de re* reading can be paraphrased as 'The number of planets is such that it is necessary of it that it is odd'. The *de dicto* reading is false, but the *de re* reading is true, given that we still count Pluto as a planet. Note that the ambiguity of these sentences no longer hinges on the occurrence of a modality and a quantifier, but here two readings are distinguished for sentences involving a modality and a term.

The following formulas capture the distinction

$$\Box\langle \lambda x.Px \rangle(c) \tag{5}$$

$$\langle \lambda x.\Box Px \rangle(c) \tag{6}$$

Formula 5 reads "necessarily c is a P" and formula 6 reads "c is necessarily a P". As was already noted by Pratt [Pra76], in first order dynamic logic, there is a close link between assignments and lambda expressions. Consider the following formulas

$$\Box\langle x := c \rangle Px \tag{7}$$

$$\langle x := c \rangle \Box Px \tag{8}$$

These are equivalent to 5 and 6 respectively. Thus the problem of *de dicto* and *de re* readings of sentences can be reduced to the question whether the denotation of a term is fixed *outside* or *inside* the scope of the modality. Using wildcard assignment we can formalize Quine's example in dynamic term-model logic with the following formulas.

$$[r]\langle x :=? \rangle Px \tag{9}$$

$$\langle x :=? \rangle [r] Px \tag{10}$$

where r refers to Ralph, and Px means x is a spy. Comparing (7) and (8) with (9) and (10) shows an attractive syntactical similarity between *de dicto* and *de re* quantification and predication in dynamic term-model logic.

The addition of wildcard assignment as a program that can be combined with other programs yields much more expressive power, than having separate quantifiers. For first-order dynamic logic Meyer and Winklmann [MW82] show that one can express that a pair of terms is linked by the transitive closure of a relation using wildcard assignment. In a similar way wildcard assignments can be used in dynamic term-modal logic to express common knowledge among the group of all agents.

$$[(x := ?; x)^*]\varphi \tag{11}$$

Wildcard assignment can best be seen as an unbounded nondeterministic choice. This means that formula (11) is true at a world w and assignment g iff φ is true at all worlds v that can be reached by a path in $(\bigcup_{a \in A} R(a))^*$. We can also express subgroup common knowledge using a unary predicate P. This predicate designates a subgroup of A. To say that it is common knowledge among the members of this subgroup that φ can be expressed as follows.

$$[(x := ?; ?Px; x)^*]\varphi \tag{12}$$

Of course the interpretation of P can change from world to world. Thus this is the notion of indexical or non-rigid common knowledge studied first by Moses and Tuttle [MT88] and Dwork and Moses [DM90]. In this language it can easily be seen that non-rigid common knowledge is closely related to the notion of relativized common knowledge as it is studied by van Benthem et al. [vBvEK06], which can be defined with the following formula.

$$[(x := ?; ?\psi; x)^*]\varphi \tag{13}$$

The ability to express non-rigid common knowledge immediately raises the question whether it is also possible to express rigid common knowledge. This would lead us to develop a second-order term-modal logic, where unary predicates P are taken to be names of groups. A unary variable would then be a rigid designator of a group and we could express that φ is common knowledge among the actual members of group P as follows.

$$[X := P][(x := ?; ?Xx; x)^*]\varphi \tag{14}$$

This would take us beyond the scope of this paper, and we will not investigate this issue further here.

4   Update models

One of the most influential ideas in dynamic epistemic logic, developed by Baltag, Moss and Solecki [BMS98], is to model *events* involving information

in the same way as *situations* involving information are modeled. Situations involving information are modeled using epistemic models containing different possible worlds, some of which the agents cannot distinguish. Events involving information can be modeled using update models containing different possible events, some of which the agents cannot distinguish.

DEFINITION 4 (update models). An update model for a finite set of agents A with a language $\mathscr{L}$ is a triple $U = (E, R, pre)$ where

- E is a finite non-empty set of events,

- $R : A \to \wp(E \times E)$ assigns an accessibility relation $R(a)$ to each agent $a \in A$,

- $pre : E \to \mathscr{L}$ assigns a precondition to each event,

In this definition the language $\mathscr{L}$ is just a parameter. Baltag et al. [BMS98] insert these models as modalities in a logical language, and the semantics of these modalities are given by a model product construction. Objections have been raised against this idea (e.g. by myself in my dissertation [Koo03, page 55]), because it seems to blur the distinction between syntax and semantics. But having model-like structures in the language is not as objectionable as it may seem. In regular expressions are modalities. As is known from Kleene's theorem, regular expressions correspond to finite state automata. So, from a formal point of view there is no need to favor graph-like structures over strings. This insight was used to translate the logical language of Baltag et al. [BMS98] to the language of (automata) PDL [KvB04, vBvEK06]. This also shows that in the case of dynamic epistemic logic one can use structures such as graphs in a logical language just as well as strings. Since update models are intuitively very appealing for reasoning about information change, there is quite an advantage in using update models,

In the context of dynamic term-modal logic adding update models as modalities to the logical language cannot be done directly, since the agents themselves are part of the update models, and these are now properly a part of the semantics. Instead, the accessibility relations are labeled with unary predicates (or generally, with formulas containing exactly one free variable), which correspond to groups of agents. By taking these formulas to be pairwise inconsistent and exhaustive, this set generates a partition of the set of all agents. The accessibility relation labeled with such a formula tells us which events the agents in that group cannot distinguish. Then we get the following update models.

DEFINITION 5 (first-order update models). A first-order update model U is a tuple (E, Φ, R, pre)

- $E \neq \emptyset$ a finite non-empty set of possible events.

- Φ is a finite set of pairwise inconsistent and jointly exhaustive formulas with free variable x.

- $R : \Phi \to \wp(E \times E)$ assigns an accessibility relation to each group of agents.

- pre is a function from E to $\mathscr{L}$.

Given a first-order update model $U = (E, \Phi, R, pre)$, an epistemic model $M = (W, A, R, I)$, a world w, and a variable assignment g, there is a unique $\varphi \in \Phi$ for each $a \in A$ such that $M, w, g[x/a] \models \varphi$. We refer to this formula as $\varphi_a^{M,w,g}$. Thus we obtain the accessibility relation in the update model for each agent, world, and assignment.

DEFINITION 6. Given a first-order update model $U = (E, \Phi, R, pre)$, an epistemic model M, a world w, and a variable assignment g, the relation $R_a^{M,w,g} \subseteq E \times E$ is defined as

$$R_a^{M,w,g} = R(\varphi_a^{M,w,g})$$

We write $R_a^{M,w,g}(e)$ for the set $\{f \mid (e, f) \in R_a^{M,w,g}(e)\}$.

Now we are ready to define the appropriate notion of product update. We have to adapt the definition proposed by Baltag et al. [BMS98]. Their idea is that the new set of possible world consists of pairs of worlds from the original epistemic model and events from the update model. An agent cannot distinguish one pair from another pair if the agent could not distinguish the worlds in the original epistemic model and if the agent cannot distinguish the events in the update model. In the case of first-order product update we have to add an extra condition. For it may be the case that an agent cannot distinguish two events but can infer to which group of agents he belongs from the mere fact that the agent gets certain information. We take the information an agent gets to be equal to the set of events accessible to that agent. Therefore, if these are different for two worlds, then the agent can distinguish these worlds.

DEFINITION 7 (First-order product update). Let an epistemic model $M = (W, A, R, I)$, an assignment g and a first-order update model be $U = (E, \Phi, R, pre)$ be given. The result of executing U in M is an epistemic model $M \otimes U = (W', A', R', I')$ such that

- $W' = \{(w, e) \mid M, w, g \models pre(e)\}$,

- $A' = A$,

- $(w, e)R_a(v, f)$ iff $wR_a v$ and $eR_a^{M,w,g}f$ and $R_a^{M,w,g}(e) = R_a^{M,v,g}(f)$.

- $I'(w, e) = I(w)$.

The first-order product update construction used in the definition of can be used to provide semantics when the logical language of dynamic term-modal logic is extended with first-order update models as modalities.

$$M, w, g \models [U, e]\varphi \quad \text{iff} \quad M, w, g \models pre(e) \text{ implies } M \otimes U, (w, e) \models \varphi$$

The semantics is best illustrated with an example. Suppose there is a very inconstant professor. He changes his mind all the time about everything, especially about which colleagues he considers to be his friends and which he considers to be his enemies. You never know to which of these groups you belong. He celebrates his birthday each year by having a party in his favorite pub or at home, but he finds it very hard to decide where to throw the party. On the day before his birthday he sends his friends the message where the party is going to be. If you are one of his friends, you either get the message "see you in the pub tomorrow" or "see you at my house tomorrow". Of course the professor does not send messages to his enemies. In one thing he is quite constant, in the sense that this goes on every year. Therefore this procedure is common knowledge among his colleagues.

The update model for the birthday invitation is given in Figure 1, where Fx means "x is a friend" and p means "the party will be in the pub". It is common knowledge that the invitation is sent, but only friends find out where the party is. Suppose that we limit our attention to two colleagues of the professor, a and b, who do not know whether the professor regards them as his friends or not, and do not know where the party will be, but do know that today is the day before the professor's birthday. The effect of the invitation is illustrated in Figure 2. A world in the epistemic model is represented with a sequence of three bits. The first represents whether a is a friend. The second represents whether b is a friend. The third represents whether the party will be in the pub or not.

Note that one of the effects of the update is that afterwards every agent knows whether he is a friend of the professor or his enemy. Even though that information is not included in the invitation, it can be inferred by the mere fact that one gets an invitation or not. This is why we needed the third condition in the definition of product update for the accessibility relation of the agent. After the update the professors' friends know where the party is,
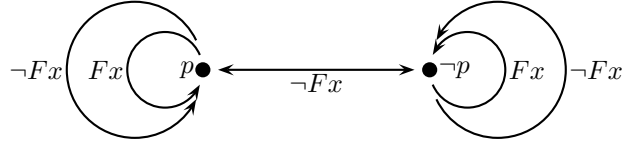
Figure 1. A first-order update model for the invitation. Formally the model is defined as $U = (E, \Phi, R, pre)$, where $W = \{e, f\}$, $\Phi = \{Fx, \neg Fx\}$, $R(Fx) = \{(e, e), (f, f)\}$, $R(\neg Fx) = E^2$, $pre(e) = p$, and $pre(f) = \neg p$. In the figure above, the event e is on the left and f is on the right. The preconditions are attached to these nodes (p on the left, and $\neg p$ on the right). An arrow labeled with a unary predicate represents an element of the accessibility relation assigned to that unary predicate.
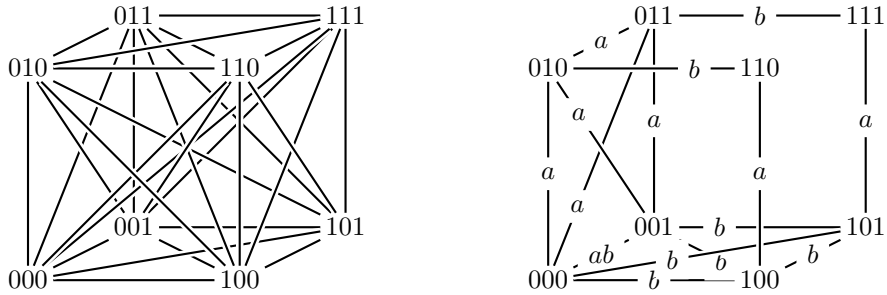

Figure 2. The epistemic models for before and after the invitation. Since, we assume that the relations are equivalence relations, the reflexive arrows are omitted and the connections are undirected. In the picture on the left a line indicates that the worlds are indistinguishable for both agents. In the picture on the right the lines are labeled with those agents for which the worlds are indistinguishable.

it is even non-rigid common knowledge among his friends where the party is, even though it is not common knowledge among his friends who his friends are.

If we compare the first-order update model to the propositional update model for the same event, we can see the advantages of using first-order update models. The propositional update model would essentially have the same structure as the epistemic model of the situation after the invitation. First-order update models thus provide a very compact description of the event.

5   Conclusion and questions for further research

Dynamic term-modal logic forms a philosophically sound basis for futher developing epistemic predicate logics. It could be further developed by allowing varying domains such that matters of existence can also be investigated from an epistemic perspective. On the technical side, it would be interesting to see whether some meta-logical results can be obtained. As was noted above, it would be hard to find an axiomatization or a decidable fragment of the language.

The update models developed in Section 4 can be studied independently from the logic developed in Section 2. They can be added as modalities to any first-order modal logic, although their epistemic interpretation seems most interesting. They provide a compact way of describing events involving information for different groups of agents and the product construction captures many of the intuitions regarding information change, including the ability to infer group membership from the information one receives.

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